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Customized Bundle Pricing for Information Goods: A Nonlinear Mixed-Integer Programming Approach

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This paper proposes using nonlinear mixed-integer programming to solve the customized bundle-pricing problem in which consumers are allowed to choose up to N goods out of a larger pool of J goods. Prior work has suggested that this mechanism has attractive features for the pricing of information and other low-marginal cost goods. Although closed-form solutions exist for this problem for certain cases of consumer preferences, many interesting scenarios cannot be easily handled without a numerical solution procedure. In this paper, we investigate the efficiency gains created by customized bundling over the alternatives of pure bundling or individual sale under different assumptions about customer preferences and firm cost structure, as well as the potential loss of efficiency caused by pricing with incomplete information about consumer reservation values. Our analysis suggests that customized bundling enhances sellers' profits and enhances welfare when consumers do not place positive values on all goods, and that this consumer characteristic is much more important than the shape of the valuation distribution in determining the optimal pricing scheme. We also find that customized bundling outperforms both pure bundling and individual sale in the presence of incomplete information, and that customized bundling still outperforms other simpler pricing schemes even when exact consumer valuations are not known ex ante.

Key words: information goods; electronic commerce; customized bundle; pricing; nonlinear programming; integer programming
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1. Introduction

The emergence and rapid growth of low-cost reproduction and distribution technologies for information goods has led researchers and information goods providers to rethink the economics of selling information goods, such as music, motion pictures, books, and news. Many information goods have long been sold in bundles (e.g., articles bundled into newspapers or magazines, and songs bundled into CDs) to economize on the production and distribution costs of physical media. However, if electronic reproduction and distribution of information goods becomes almost costless, then the cost benefits associated with bundling are greatly reduced. As technological progress makes detailed monitoring, customized distribution, and micropayment possible, some scholars and practitioners have predicted that individual sales of information goods would become more popular. However, even if distribution costs no longer motivate bundling, Bakos and Brynjolfsson (1999) have shown that a multiproduct monopolist can garner greater profits by bundling a large number of goods and selling them for a fixed price (a "pure bundling" strategy) when the marginal cost of bundling an additional good is low (preferably zero) and the value of each good is identically distributed. Under these conditions, the value of a bundle (per good) converges to a constant as the bundle becomes large by the law of large numbers, enabling a monopolist to precisely determine consumers' willingness to pay for the bundle, which maximizes profits and minimizes consumer surplus. At the same time, this pure bundling strategy can also minimize deadweight loss, which makes it socially efficient, and computing the single optimal bundle price is a straightforward problem with complexity that scales linearly with the number of goods.

However, there are some practical difficulties associated with this pure bundling strategy. First, the "law of large numbers" requirement works best if goods have zero marginal cost. If, on the other hand, the marginal cost is positive, then the total cost of a sufficiently large bundle can itself become large, negating the benefits of bundling. For instance, iTunes purports to have over 10 million songs in its library that it is licensed to distribute at an approximate marginal cost of $0.50 (mostly royalties to copyright owners plus some small operational cost). This marginal cost would require a pure bundle price exceeding $5 million to be profitable. If the marginal cost is nonnegligible and the number of items is high, then any feasible bundling solution must only include a subset of the available goods. However, partial pure bundles can create substantial deadweight loss (see Hitt and Chen 2005).

Second, the condition of identical distributions across consumers (or goods) may also be violated in many ways, including very simple and reasonable assumptions about consumer preferences such as different budget constraints, or if consumers have different numbers of goods they value positively. Under these conditions, the variance of consumer valuation does not necessarily decrease as more goods are bundled. This could explain why most online music distribution uses a per-song price. In addition, if the differences across consumer groups are large, an information goods provider may be forced to leave some consumers with positive bundle valuations unserved to extract more surplus from other high-value consumers, leading to high deadweight loss. The general problem is that pure bundles are not suited to handle consumer heterogeneity because pure bundles only provide a single pricing instrument—namely, the price of the entire pool of goods.

Perhaps the simplest example that violates the "identically distributed" assumption is budget constraints. For example, extending the iTunes example above, consider a world in which there are two types of consumers: casual music buyers who spend $100 per year on music, and music aficionados who are willing to spend $1,000 per year. Because the pure bundling framework lacks any notion of budget constraints, one way that these types of preferences could be accommodated is if there were different valuations for each good for each type of consumer. However, the efficiency of pure bundling is no longer guaranteed, because it can be desirable for the monopolist to exclude the entire casual music segment to avoid leaving the aficionados with large amounts of surplus (this outcome is optimal if more than 10% of the consumer populations are aficionados if we are constrained to pure bundling).

Another simple way in which the identically distributed assumption can be violated is if consumers have differing numbers of goods they value positively. This example appears in the current debate about cable television unbundling. In its investigation of pricing practices in cable TV, the Federal Communications Commission (FCC) cites a number of studies that reveal that consumers value only a small subset of channels and that the number and identity of channels preferred varies considerably across consumers. However, most cable TV systems essentially offer a pure bundling approach, with the exception of a few "premium" channels such as HBO that can be purchased separately (referred to as "a la carte pricing" in an FCC study, and "individual sale" in our analysis). Because advanced cable decoder boxes enable individual pricing of cable channels, the FCC has been investigating the potential for complete unbundling (i.e., a la carte pricing) and has argued that the unbundled model would increase consumer surplus, although the industry offers a contrasting opinion. Indeed, if only pure bundling and individual sale are considered, then it is simply a choice between two obviously inefficient systems: pure bundling ignores differences across consumers and pure unbundling creates efficiency losses because price exceeds marginal cost. At this level, it simply becomes an accounting issue (about which inefficient system is better), but one heavily constrained by the lack of suitable data upon which to make the decision.

In an effort to find a "middle ground" that both preserves the benefits of pure bundling and offers the flexibility of individual sale, Hitt and Chen (2005) analyzed the concept of customized bundling, a pricing mechanism whereby consumers may select a fixed number of goods (which they choose) (N) out of the total goods available (J) for a fixed price (P).[1] This scheme has a number of desirable properties: Compared to a la carte pricing, it is both more profit enhancing and satisfies more consumer needs (from variance reductions through bundling); compared to pure bundling, it is more flexible and efficient because it allows more than one price point to accommodate different consumer segments. Hitt and Chen (2005) showed that under moderate marginal cost, the optimal customized bundle size can be interior, involving neither an individual sale nor full bundling.

[1] The term "customized bundling" is attributed to Hitt and Chen (2005). However, there have been several prior studies with analogous bundling schemes (e.g., Chen 1998, Mackie-Mason et al. 2000).

However, although Hitt and Chen (2005) provide a characterization of the general properties of customized bundling, their analysis utilized an analytical approach that limited the types of settings that could be considered. Analysis of customized bundling involves computing sums of order statistics, which is not possible in closed form except for selected probability distributions. In addition, general properties of customized bundling can only be derived under additional assumptions common in the nonlinear pricing and multidimensional screening literature, such as the Spence-Mirrlees single-crossing property.[2] Although these assumptions are convenient for analytical work, there is no guarantee that they hold in practice. Furthermore, it is considerably more difficult in their analytical model to study conditions in which sellers may be constrained to simultaneously offer different pricing schemes that may not be optimal (e.g., selling CDs for \$10 while allowing individual songs to be purchased for \$1). Finally, customized bundling is likely to be more difficult to implement in practice than either of the simpler schemes, making it important to understand the conditions under which using customized bundling creates significant gains over the simpler alternatives.

In this paper, we extend the Hitt and Chen (2005) work and explore the properties of customized bundling using a nonlinear mixed-integer programming approach. This approach allows us to accommodate any set of assumptions about good valuations and customer heterogeneity. First, we propose a computationally efficient method for solving customized bundling problems. Second, we demonstrate that this method has the expected properties by comparing numerical results with theoretical solutions in settings in which the properties of customized bundling are known or easily derived in closed form. Finally, we investigate the performance of customized bundling in a variety of settings that are not analytically tractable, such as settings with a large number of goods or consumer types. This enables us to consider issues such as the following:

1. How much efficiency gain does customized bundling create over the alternatives of pure bundling or individual sale under different assumptions of consumer preference?

2. How well does customized bundling perform, relative to the alternatives, if consumers have heterogeneous preferences? How much does customized bundling improve total welfare under these conditions?

3. How costly is incomplete information about consumer reservation values (the mixed bundling literature often assumes complete information, but that is unlikely to hold in practice) rather than just the distribution of values for different goods for different consumers?

Overall, our results indicate that the number of positively valued goods is more important than the customer valuation function in determining the optimal pricing scheme. Regardless of customer valuation functions, as long as customers differ in the number of goods they positively value, customized bundling dominates pure bundling and individual sale and enhances welfare. This finding has interesting strategic implications, because the information about the number of goods positively valued is relatively easier to obtain than customer valuation functions. It is much easier to ask whether a customer positively values a good than to ask her to quantify the value she obtains from the consumption of a good. Our results also show that pure bundling and individual sale underperform more in the presence of incomplete information, and that customized bundling still outperforms other simpler pricing schemes even if exact consumer valuations are not known ex ante.

In §2, we review the existing literature on bundling and bundle-pricing algorithms. In §3, we present our model formulation and solution approach (details of the solution procedure appear in the online supplement, provided in the e-companion).[3] Numerical results and case analyses are presented in §4, followed by discussion and concluding remarks in §5.

2. Literature Review

The literature on bundling often considers three forms: pure bundling, unbundling, and mixed bundling (Adams and Yellen 1976, Stremersch and Tellis 2002). *Pure bundling* is a strategy in which a firm sells only the bundle and not the products separately. *Unbundling, or individual sale*, is a strategy in which a firm only sells the products separately. *Mixed bundling*, on the other hand, refers to a strategy in which a firm sells both the bundle and each of the products in the bundle separately. A more complex mixed-bundling strategy is when a firm, in addition to selling each product separately and the full bundle, also sells other bundles that consist of different subsets of products (which is often termed "the *full mixed-bundling* problem").

The advantage of bundling can be traced back to Stigler (1963) who observed that bundling can be profitable if consumers' willingness to pay for two

[2] Essentially, when combined with free disposal, this means that consumers have a strict ordering in their demand levels across the entire consumption space. Some consumers have strictly higher willingness to pay for any given number of products than others, and this ordering is preserved over all bundle sizes.

[3] An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

goods is negatively correlated. A number of subsequent papers found that, under reasonable assumptions, offering both a two-good bundle and the individual items ("mixed bundling") together is the best for the two-good case (e.g., Adams and Yellen 1976, Schmalensee 1984, McAfee et al. 1989, Salinger 1995). Venkatesh and Kamakura (2003) showed that these insights extend to cases for which the two goods are complements or substitutes.

The mixed-bundle pricing problem for more than two goods is considerably more challenging. Economists have tried to identify conditions for which closed-form solutions may be feasible or have made certain assumptions to help keep the analyses tractable. Spence (1980) was one of the first analyses to clearly formulate the multiproduct case and identified some special cases under which problems can be solved in closed form. Extending this work, McAfee and McMillan (1988) found feasible solutions in the case of linear utility, whereas others assumed that the valuations for different customers can be ordered in specific ways or satisfy certain separability conditions (e.g., Armstrong 1996, Sibley and Srinagesh 1997, Armstrong and Rochet 1999). However, because the complexity of the problem increases dramatically as more goods are considered, it is generally hard to consider bundling problems with large numbers of items. Moreover, although interesting insights can be obtained with the common assumptions that lead to tractability, the literature has paid little attention to the cases in which these conditions may not hold, and it is not clear what happens under these cases.

Recently, researchers have begun to consider pricing for large-number bundling problems, especially for information products that are presumed to have very low marginal costs. Bakos and Brynjolfsson (1999) showed that when marginal costs are negligible and customers have identically distributed valuations, pure bundling of large numbers of goods is optimal. Chung and Rao (2003) also focused on the pure bundling case and developed a product attribute model of consumer utility for bundling. Jedidi et al. (2003) studied the bundling strategies for goods in which values may be related. Chuang and Sirbu (1999), on the other hand, showed that offering pure bundling along with selling each good independently ("mixed bundling") can dominate either pure bundling or individual sale alone.

Another stream in this literature focused on directly utilizing numerical methods to determine the full mixed-bundling solution. Hanson and Martin (1990) formulated the problem as a mixed-integer programming problem in which a firm was attempting to find the optimal price for each possible bundle, given known values for each good and each potential bundle. Because the complexity of their model setup grows exponentially with the number of goods, they were only able to obtain solutions for up to $J = 21$ possible goods. In addition to the size limitation, which renders the full mixed-bundle problem infeasible for many information goods settings (e.g., $J = 10$ million for iTunes), their analysis relies on full information about consumer valuations for each product and also for each potential bundle—information that is not likely to be easily obtained. Our analysis builds on their work, adopting a mixed-integer programming approach, but simplifying the setup to handle much larger problems by imposing the structure of *customized bundling* (as defined in Hitt and Chen 2005), which is a strategy in which the price is dependent only on the number of goods in the bundle, but not on the specific content of the bundle. In customized bundling, consumers pay for the right to choose any N out of J total goods. This differs from the full mixed-bundling problem, in which the firm decides not only the price for each bundle but also the content to be included in each bundle. This transforms the problem from exponential to linear in size. This tractability advantage becomes important when the number of goods is large. In addition, this improvement in tractability enables us to consider a wider range of conditions on consumer preferences and to revisit the assumption that the monopolist has perfect information about customer valuations by examining how well our algorithm performs if those valuations are not known with certainty by the seller. Our principal contribution in this work is to provide a much broader characterization of the properties of the customized bundling approach by using numerical optimization method. Our objective is to provide a richer characterization of the structure of the customized bundling problem given its relatively recent introduction into the bundling literature and the observation that customized bundling has not yet received as thorough analysis as mixed bundling, despite its potential utility.

3. Problem Formulation and Solution Approach

In this section, we formulate the optimal bundling and pricing problem for an information goods provider that distributes J goods to I consumers. The model is developed from the seller's perspective. The problem for the seller is to decide how many goods to be included in each bundle, and how to price these bundles to maximize her profit, subject to a set of consumer participation and incentive compatibility constraints. Following Stigler (1963), we first assume that customer demand information is captured by a vector of reservation prices of the items that go into a bundle that for our purposes may be fixed or generated by some sort of valuation distribution. We also assume that customers maximize consumer surplus based on the difference between the total reservation price for items in a bundle and the price they pay. The seller has to account for consumers' optimal choice behavior given their preferences and the price-product offerings, which appear as constraints in the seller's optimization problem.

Note that under the customized bundling strategy the price is only determined by the size of the bundle and not by the bundle contents. For example, the seller might decide to bundle any three CDs for \$20.00, and the buyer would choose which three CDs from a longer menu of options to buy. Different buyers may choose different CDs to purchase. In practice, there may be an overhead cost for offering different product bundles. This overhead cost may arise due to the need for the provider to maintain a price list with several consumer choices. Offering more bundles in a sales menu may imply more complicated operations, and therefore higher overhead costs. In addition, consumers may also incur high cognitive costs in evaluating large sets of offers (Shugan 1980), which may indirectly influence consumer utility. We capture the costs associated with maintaining multiple choices as a "menu cost" (Wu and Chen 2007). As a result, an information goods provider faces a trade-off between offering more choices (which captures more value) and incurring greater menu costs for a larger choice set.

Table 1 outlines the basic notation we use to formulate customized bundling as a nonlinear mixed-integer programming problem. The primal problem is given by IP.

Primal Problem IP:

$$\max_{P_j,\,S_i,\,X_{ij},\,Y_j} \sum_{i=1,\dots,I}\sum_{j=1,\dots,J}(P_j-B_j)X_{ij}-\sum_{j=1,\dots,J}MY_j \tag{1}$$

$$\text{s.t.}\quad S_i\ge(R_{ij}-P_j)Y_j,\quad i=1,\dots,I;\ j=1,\dots,J \tag{2}$$

$$S_i=\sum_{j=1,\dots,J}(R_{ij}-P_j)X_{ij},\quad i=1,\dots,I \tag{3}$$

$$(R_{ij}-P_j)X_{ij}\ge 0,\quad i=1,\dots,I;\ j=1,\dots,J \tag{4}$$

$$\sum_{j=1,\dots,J}X_{ij}\le 1,\quad i=1,\dots,I \tag{5}$$

$$X_{ij}\le Y_j,\quad i=1,\dots,I;\ j=1,\dots,J \tag{6}$$

$$S_i\ge 0,\quad i=1,\dots,I \tag{7}$$

$$P_j\ge 0,\quad j=1,\dots,J \tag{8}$$

$$X_{ij}=0 \text{ or } 1,\quad i=1,\dots,I;\ j=1,\dots,J \tag{9}$$

$$Y_j=0 \text{ or } 1,\quad j=1,\dots,J. \tag{10}$$

Table 1 Definitions of the Parameters and Variables Used in the Model

Given parameters	
B_j:	Bundle cost of creating a bundle of j goods. This may include the sum of marginal production cost, distribution cost, transaction cost, any binding cost, etc. We assume this is the same for any kind of bundles of j goods.
I:	There are total I potential customers in our target market.
J:	The vendor has total J different kinds of information goods in hand.
M:	Marginal menu cost if we add one more bundle choice on the menu.
V_{ik}:	Customer i's reservation price for his kth favorite information goods.
R_{ij}:	Total reservation price of customer i's top j favorite goods, i.e., $R_{ij}=\sum_{k=1,\dots,j}V_{ik}$.
Decision variables or intermediate variables	
P_j:	The price assigned to the bundle of j goods.
S_i:	Consumer surplus for customer i.
X_{ij}:	The decision variable which is one if consumer i chooses to buy the bundle of j goods, and zero otherwise.
Y_j:	The decision variable which is one if the vendor chooses to offer the bundle of j goods on the menu, and zero otherwise. ($\sum_{j=1,\dots,J}Y_j=$ no. of customized bundles offered)

The objective function (1) maximizes the total profits of the vendor. This is calculated by summing the profit obtained from each customer minus the menu cost of the vendor. Each constraint is explained as follows.

Constraint (2) ensures that each customer maximizes her surplus S_i in making her choice. This is achieved by requiring that the final consumer surplus obtained from her choice of bundle is no less than the consumer surplus from any other bundle offered by the seller (these are the incentive compatibility constraints). Constraint (3) defines consumer surplus as the difference between customer i's reservation price and the market price of the bundle she chooses. Constraint (4) ensures that the consumer will choose a bundle only if her surplus on this bundle is nonnegative (these are the individual rationality constraints); otherwise, she will not choose this bundle. In other words, if $P_j>R_{ij}$, then X_{ij} must be zero. Constraint (5) ensures that each customer will purchase exactly one bundle, or will not make a purchase at all.[4] This constraint could be relaxed and this relaxation should further favor the customized bundle setting. Constraint (6) ensures that only if the vendor offers the bundle of j goods can customers choose this kind of bundle; otherwise, no such choice is available. Constraints (7) and (8) are nonnegativity constraints for consumer surplus and bundle price.[5] Constraint (9) enforces the integer property of

[4] This is a common assumption adopted in the bundling literature, both for the analytical approach and optimization approach (see, e.g., Spence 1980, Hanson and Martin 1990, Hitt and Chen 2005).

[5] Note that constraint (7) is redundant and can be removed from the model (as we later do).

the decision variables with respect to consumer purchases, and constraint (10) enforces the integer property of the decision variables with respect to bundle offerings.

We design an approach using Lagrangian relaxation and subgradient methods to solve this complicated nonlinear mixed-integer program for cases in which the number of potential items that can be bundled is large. Due to the space constraints, we briefly describe this approach, and a more detailed version of this solution approach is provided as an online supplement to the paper for interested readers.

Solution Approach. By using the Lagrangian relaxation method, we can transform the primal problem (IP) mentioned above into the following Lagrangian relaxation problem (LR) where constraint (5) is dualized.

Problem LR:

$$\phi(a) = \max_{P_j, S_i, X_{ij}, Y_j} \sum_{i=1,\dots,I} \sum_{j=1,\dots,J} (P_j - B_j) X_{ij} - \sum_{j=1,\dots,J} MY_j + \sum_{i=1,\dots,I} a_i \left(1 - \sum_{j=1,\dots,J} X_{ij}\right) \tag{11}$$

$$\text{s.t.} \quad S_i \geq (R_{ij} - P_j) Y_j, \quad i = 1, \dots, I;\ j = 1, \dots, J \tag{12}$$

$$S_i = \sum_{j=1,\dots,J} (R_{ij} - P_j) X_{ij}, \quad i = 1, \dots, I \tag{13}$$

$$(R_{ij} - P_j) X_{ij} \geq 0, \quad i = 1, \dots, I;\ j = 1, \dots, J \tag{14}$$

$$X_{ij} \leq Y_j, \quad i = 1, \dots, I;\ j = 1, \dots, J \tag{15}$$

$$S_i \geq 0, \quad i = 1, \dots, I \tag{16}$$

$$P_j \geq 0, \quad j = 1, \dots, J \tag{17}$$

$$X_{ij} = 0 \text{ or } 1, \quad i = 1, \dots, I;\ j = 1, \dots, J \tag{18}$$

$$Y_j = 0 \text{ or } 1, \quad j = 1, \dots, J. \tag{19}$$

This nonlinear integer programming problem *LR* is complex and hard to solve. However, it is amenable to simplification. First, given constraints (13) and (14), constraint (16) is redundant, so we can remove it. Second, constraint (13) simply defines the consumer surplus. If we replace S_i in constraint (12) with $\sum_{k=1,\dots,J} (R_{ik} - P_k) X_{ik}$, we can remove constraint (13). Also, note that *LR* can be decomposed into J different problems except for the complicated constraint (12). Unfortunately, (12) cannot be dualized because the resulting Lagrangian objective function will have the nonlinear terms of $P_j X_{ij}$, $P_k X_{ik}$, and $P_j Y_j$ as well as the double summation over J, which makes it impossible to decompose the problem further into J different subproblems. As a result, we decide to drop constraint (12) and solve a relaxed version of *LR*, which we call *Problem R*.

Although we could not directly solve the Lagrangian relaxation problem, *Problem LR*, we can successfully solve *Problem R* with the algorithm we design. According to the weak Lagrangian duality theorem, the optimal objective value of *Problem LR* is an upper bound of the optimal objective value of *Primal Problem IP*. Also, because *Problem R* is a relaxation of *Problem LR*, the optimal objective value of *Problem R* is also an upper bound of the optimal objective value of *IP*. We then construct the following dual problem to calculate the tightest upper bound and solve the dual problem by using the subgradient method:

$$\min \ V(a) \quad \text{(D)}$$
$$\text{s.t.} \ a \geq 0.$$

where $V(a)$ is *Problem R*.

After the implementation of the subgradient optimization procedure, although we did not observe a point with zero subgradient, we have an upper bound on the optimal objective value of the primal problem. However, as expected, no primal feasible solution was found in the process due to the structure and complexity of this bundle-pricing problem because the subdivision into J independent subproblems means that constraint (5) is likely to be violated. Therefore, we try to utilize the Lagrangian solutions to develop a heuristic algorithm to get the primal feasible solutions.

Algorithm "Bundling."

Step 1. For each potential customer i, choose the bundle k (only those with $Y_j = 1$) with the largest positive surplus. Set X_{ik} to one and other X_{ij} to zero. (This enforces constraints (12) and (5).)

Step 2. For each offered bundle (those with $Y_j = 1$), calculate its revenue achieved $\sum_{i=1,\dots,I} (P_j - B_j) X_{ij}$. Choose the bundle k with the smallest revenue. If the revenue is smaller than M, we do not offer this bundle by setting the corresponding P_j, X_{ij}, and Y_j to zero and go to step 1. (The rationale behind this is that based on our primal objective function (1), it is reasonable for us to offer the bundle only when the revenue from the bundle could cover the fixed cost M.)

Step 3. Calculate consumer surplus S_i according to constraint (3).

Further details about Lagrangian relaxation and subgradient methods can be found in Fisher (1981) and Nowak (2005). The computational results based on this approach are presented in the following section.

4. Analysis and Results

Bakos and Brynjolfsson (1999) theoretically showed that if customer valuations are independently and identically distributed (i.i.d.) and goods have very low (or zero) marginal costs, then pure bundling is the optimal strategy. Hitt and Chen (2005) showed that customized bundling can be optimal and derives a closed-form solution under certain conditions including random valuation with moderate marginal costs, and a special class of valuation functions (originally proposed by Chuang and Sirbu 1999) in which customers have a common functional form over their rank order of goods, but differ in the total number of goods with positive value. In our analysis, we are no longer constrained to use analytically tractable assumptions, so we can potentially explore more realistic settings in examining the optimal structure of the customized bundling solution and its relative performance gains (profit, consumer surplus, and social welfare) over alternative pricing approaches.

We consider several specific forms of customer heterogeneity. First, following prior work, we assume that consumers may differ on *the number of goods they positively value* (captured by the parameter k). A number of empirical studies have suggested that k varies across consumers (e.g., King and Griffiths 1995). Second, we assume that consumers can vary in their *distribution of value over goods*, both in terms of the functional form (e.g., uniform, exponential) and the parameters of these distributions (e.g., exponential distributions with different means). Note that by changing valuation assumptions we also implicitly assume that consumers may have different overall budget constraints.

Another advantage of a numerical approach is that we can examine the performance of customized bundling under varying information conditions for the seller. For instance, Hanson and Martin (1990) assume that consumers' reservation prices are known to the seller. Hitt and Chen (2005) and Bakos and Brynjolfsson (1999) rely on exact structures of distribution functions and large numbers of consumers (such that the theoretical and empirical distributions are assumed to be indistinguishable). Here, we can examine the importance of knowing the exact value of a customer's reservation price versus knowing the distribution of reservation prices. This may be useful in determining the amount of information needed to utilize these different pricing approaches in practice.

4.1. Replicating Prior Theoretical Results

In this section, we first examine the efficiency gains created by customized bundling versus the alternatives of pure bundling or individual sale under different assumptions of consumer preferences. To study this, we assume all consumers have the same preferences and compare the performance of different pricing schemes by varying the parameters for consumer valuation distributions: number of goods positively valued (k) and the functional form of the distribution. Note that for goods not positively valued by customers, we assume their values are zero.[6] To compare different pricing schemes, we first set marginal menu cost M and marginal bundle cost B_j as zero, because these are the settings normally considered in the prior bundling literature. We consider variations in marginal bundle cost and menu cost in a later analysis.

Table 2 **Replicating Optimal Pricing Strategy for Homogeneous ("Single-Type") Customers**

	$k=20$	$k=60$	$k=100$	$k=20$	$k=60$	$k=100$
Customer's valuation for goods	U(0, 2)	U(0, 2)	U(0, 2)	Exp(1)	Exp(1)	Exp(1)
No. of potential customers I	100	100	100	100	100	100
No. of goods J	100	100	100	100	100	100
Avg. no. of customized bundles offered	1	2	6	1	2	5
Avg. best customized bundling profit found	1,527.8	5,097.3	8,719.3	1,339.9	4,650.4	8,084.1
Avg. best pure bundling profit	1,526.5	5,096.7	8,722.1	1,339.0	4,652.7	8,089.5
Avg. best individual sale profit	1,005.7	3,008.6	5,005.9	745.2	2,216.9	3,697.0
Avg. profit improvement from pure bundling to customized bundling (%)	0.1	0.0	−0.0	0.1	−0.0	−0.1
Avg. profit improvement from individual sale to customized bundling (%)	52.0	69.4	74.2	79.9	109.8	118.7
Avg. duality gap (%)	24.1	16.4	12.2	34.5	24.0	19.2
Avg. computational time	<1 min	<1 min	<1 min	<1 min	<1 min	<1 min

Table 2 presents the first of several numerical experiments. We compute average profits under three pricing schemes: individual sale, pure bundling, and customized bundling. We also compare the average relative profits of the three pricing schemes as well as the average duality gaps and computational time. Hereafter, we will use the notation U(a, b) to describe a uniform distribution over the interval $[a, b]$, and Exp(λ) to describe an exponential distribution with mean λ.

If all consumers have the same preferences (in terms of demand distribution and number of goods valued positively), this is equivalent to having a single "type" of consumer. Bakos and Brynjolfsson (1999) show that if consumers value all goods positively, then pure bundling is optimal. In fact, as long as there is zero marginal cost, this result could be extended to the case in which the number of goods valued positively (k) is less than all goods because there is only a single consumer type and the monopolist is indifferent between offering a customized bundling of size (k) versus a full bundle. We therefore expect customized bundling and pure bundling

[6] This is equivalent to an assumption of free disposal (as in Bakos and Brynjolfsson 1999). Note that when consumers can derive negative values from goods they don't positively value, the performance of customized bundling will be even better than that of pure bundling.

to be approximately the same and both schemes to significantly outperform individual sale.

Table 2 confirms these intuitions by presenting best solutions found under two different demand distributions (uniform and exponential) and three values for k (20, 60, and 100) in a setting with 100 goods, 100 consumers, and zero marginal costs. For each case, 30 instances are used to get the average results.[7] Our numerical solution of the customized bundling problem yields results almost identical to those of the pure bundling problem (within 0.1%). Although there is some fluctuation due to the nature of the numerical solutions, and the fact that valuations represent realizations of random draws as well as the convergence behavior of our heuristic algorithm, the differences in the numerical results for pure bundling versus customized bundling would likely decrease if we utilized more consumers and a larger number of iterations.[8] Bundling outperforms individual sale in all cases, with greater outperformance for cases with greater numbers of positively valued goods and greater asymmetries in the valuation distribution. This is also intuitive because greater variation in demand across goods gives rise to greater deadweight loss of offering a single price. Our conclusion from Table 2 is that our algorithm appears to properly replicate the theoretically optimal solution under well-understood conditions.

For all experiments in our paper, because the solutions we got for customized bundling are actually based on a heuristic, there are still some gaps between the tightest upper and lower bounds. The duality gaps we observed tend to be data dependent. On one hand, the causes of these gaps could be simply due to the properties of the problem we face or because of deficiencies in the heuristic. On the other hand, because we already know that pure bundling is analytically proven to be optimal for the i.i.d. cases, these single-type cases also, in some way, demonstrate the quality of our solutions as well as the algorithm we proposed because our algorithm is clearly near-optimal. Given that we are able to numerically replicate the optimal solution that is analytically proven for these cases, the observed gaps suggest that the duality gaps are naturally large for our problem domain. We would also like to point out that these gaps should not affect our central conclusion that customized bundling could be profit enhancing because the numerical solutions we got are already much better than other simpler pricing schemes in many other cases. In fact, an optimal solution can only reinforce this conclusion because profit level can only increase in the optimal solution.

Observation 1. Regardless of the valuation functions and regardless of how many goods are positively valued, if the marginal cost of each good is zero and all customers have the same k and valuation function, then pure bundling is still the optimal solution even though not all goods are positively valued.

This observation is interesting, because it extends prior literature on pure bundling by showing the optimality of pure bundling to settings not considered before, such as when consumers do not positively value every good.

4.2. The Impact of Customer Heterogeneity on Optimal Pricing Scheme

We now consider how the optimal pricing scheme may change when there are different types of consumers. In general, it is difficult to obtain closed-form solutions for bundling problems with more than two consumer types without imposing auxiliary assumptions such as the single-crossing property. One major contribution of our model is that it can accommodate multiple types of customers without any assumptions on customer valuations. In this subsection, we first consider the case in which there are multiple types of customers that differ on the number of goods they will positively value, but these values are drawn from the same value distribution.

In Table 3, we hold the distribution fixed (uniform) and present results for between two and five different

Table 3 Heterogeneous Customer Types: Different k, Identical Value Distribution

	$k_1=20$, $k_2=40$	$k_1=20$, $k_2=40$, $k_3=60$	$k_1=20$, $k_2=40$, $k_3=60$, $k_4=80$	$k_1=20$, $k_2=40$, $k_3=60$, $k_4=80$, $k_5=100$
Customer's valuation for goods	U(0, 2)	U(0, 2)	U(0, 2)	U(0, 2)
No. of potential customers I	200	300	400	500
No. of goods J	100	100	100	100
Avg. no. of customized bundles offered	3	4	5	13
Avg. best customized bundling profit found	4,009.5	7,282.7	11,457.9	18,156.2
Avg. best pure bundling profit	3,301.2	6,773.7	10,505.4	15,893.0
Avg. best individual sale profit	2,997.1	6,013.6	10,033.8	15,059.7
Avg. profit improvement from pure bundling to customized bundling (%)	21.5	7.5	9.1	14.2
Avg. profit improvement from individual sale to customized bundling (%)	33.8	21.1	14.2	20.6
Avg. duality gap (%)	36.2	40.9	43.7	40.0
Avg. computational time	<1 min	<1 min	1 min	2 mins

Notes. Subscript denotes customer type. For example, $k_1 = 20$ means customer type 1 positively values 20 goods out of 100 goods.

[7] All experiments are performed on an IBM X31 notebook running Microsoft® Windows XP SP2 with Intel Pentium M 1.5 GHz processor and 512 MB RAM. The code is written in ANSI C and is compiled by Microsoft® Visual C++ 2005 Express Edition.

[8] Note that due to the same reasons, the profits we get for pure bundling are less than theoretical profits suggested by Bakos and Brynjolfsson (1999), which depends on the law of large numbers.

Table 4 Heterogeneous Customer Types: Identical k, Different Value Distributions

	U(0, 1) U(1, 2)	U(0, 1) U(1, 2) U(2, 3)	U(0, 1) U(1, 2) U(2, 3) U(3, 4)	Exp(1) Exp(2)	Exp(1) Exp(2) Exp(3)	Exp(1) Exp(2) Exp(3) Exp(4)
No. of potential customers I	200	300	400	200	300	400
No. of goods J	100	100	100	100	100	100
No. of positive values k	50	50	50	50	50	50
Avg. no. of customized bundles offered	4	4	5	4	7	8
Avg. best customized bundling profit found	6,884.1	13,868.7	23,600.3	7,993.3	16,023.2	25,019.8
Avg. best pure bundling profit	7,006.0	14,109.1	24,053.8	7,994.9	16,137.5	25,196.6
Avg. best individual sale profit	5,012.9	11,275.3	20,033.0	5,226.5	10,222.4	16,795.9
Avg. profit improvement from pure bundling to customized bundling (%)	−1.7	−1.7	−1.9	−0.0	−0.7	−0.7
Avg. profit improvement from individual sale to customized bundling (%)	37.3	23.0	17.8	53.0	56.8	49.0
Avg. duality gap (%)	35.4	41.9	45.1	48.7	48.2	51.3
Avg. computational time	<1 min	<1 min	<1 min	<1 min	<1 min	1 min

consumer segments (in which each segment is characterized by different values of k). To preserve the similarity to the results in Table 2, we consider a set of 100 goods and include 100 consumers of each type (that is, if we have two values of k, we model 200 consumers). Again, for each case, we average the results of 30 instances. We can clearly see from multiple-type cases of Table 3 that when customers are diversified by a different number of positively valued goods k but the same value distribution, there is significant profit improvement from adopting the customized bundling strategy. Pure bundling implies offering a single bundle, which means that the prices offered for the entire bundle are kept relatively low to attract all consumers (higher volume, lower price) or the firm will just target high-end customers only (higher price, lower volume). For customized bundling, multiple bundles can be offered, which means that fewer customers need to be excluded from the market.

We also test the cases in which customer types are characterized by different customer valuation functions only. In other words, we fix k for all customers, and draw the valuations for different consumer types from different value distributions. Interestingly, as shown in Table 4, when customer types are characterized by different value distributions but identical k, our algorithm fails to find better solutions that improve over pure bundling. However, if we combine the diversity of k and diversity of value distributions to the customer types (like those cases in Table 5, different k and different value distributions), customized bundling strategy again outperforms pure bundling strategy and individual sale. This suggests that heterogeneity in k plays a critical role in the performance

Table 5 Heterogeneous Customer Types: Different k and Different Value Distributions

	$k_1=20$ $k_2=40$ U(4, 5) U(3, 4)	$k_1=20$ $k_2=40$ $k_3=60$ U(4, 5) U(3, 4) U(2, 3)	$k_1=20$ $k_2=40$ $k_3=60$ $k_4=80$ U(4, 5) U(3, 4) U(2, 3) U(1, 2)	$k_1=20$ $k_2=40$ Exp(5) Exp(4)	$k_1=20$ $k_2=40$ $k_3=60$ Exp(5) Exp(4) Exp(3)	$k_1=20$ $k_2=40$ $k_3=60$ $k_4=80$ Exp(5) Exp(4) Exp(3) Exp(2)
No. of potential customers I	200	300	400	200	300	400
No. of goods J	100	100	100	100	100	100
Avg. no. of customized bundles offered	4	4	3	2	4	6
Avg. best customized bundling profit found	19,221.7	30,994.6	40,371.5	16,984.2	29,075.5	41,335.4
Avg. best pure bundling profit	17,406.3	27,142.4	34,892.4	15,189.7	26,259.2	38,672.7
Avg. best individual sale profit	18,002.0	24,025.5	24,508.8	9,605.4	15,954.5	21,044.0
Avg. profit improvement from pure bundling to customized bundling (%)	10.4	14.2	15.7	11.8	10.7	6.9
Avg. profit improvement from individual sale to customized bundling (%)	6.8	29.0	64.7	76.8	82.2	96.4
Avg. duality gap (%)	18.9	23.2	22.5	37.3	36.0	32.6
Avg. computational time	<1 min	<1 min	1 min	1 min	1 min	1 min

of the customized bundling strategy. Differences in k enable clear separation in customer types, and therefore support price discrimination with different bundle sizes, whereas the same is not true for differences in distribution in which separation of types through nonlinear pricing is less feasible. As shown in King and Griffiths (1995), people usually have different preferences over the number of goods positively valued (leading to differences in k in our setting). For all consumer populations in which this is the case, customized bundling strategy would dominate pure bundling strategy and individual sale regardless of whether the values are drawn from the same or different distributions. This finding also has interesting strategic implications, because the information about k is relatively easier to obtain. It is much easier to ask whether a customer positively values a good, or how many goods they would normally purchase, than to ask for exact valuations of each good.

To further examine the role of k, we combine a randomly drawn k with randomly drawn valuations of each good, and repeat the comparisons of different pricing schemes shown earlier. Specifically, each customer first draws a value of k from a distribution, and then draws k numbers from a valuation distribution to represent her reservation prices for the goods she positively values.

We report 12 different cases here. For each of these cases, we change the number of potential customers I and the total number of goods J. Without loss of generality, we set the number of goods J as 50% of the number of potential customers I. These 12 different cases are shown in Tables 6 and 7. For cases 1 to 6 in Table 6, we randomly pick an integer k between 1 and J (drawn from a uniform distribution) for each customer i. We then generate k random numbers out of $U(0, 2)$ for cases 1 to 3, and from Exp(1) for cases 4 to 6 to represent these positive values. Again, for the remaining goods, we assume the customer assigns a zero value to them.

For cases 7 to 12, we consider a different distribution for k. An empirical study performed by King and Griffiths (1995) indicates that out of the 80 to 100 articles in an average journal, over 40% of the readers read no more than five articles and only very few readers read more than a half of all articles in the journal. This suggests that the majority of readers have small ks. This finding can be approximated by the Poisson distribution with a small mean, and we also report cases in which k is drawn from a Poisson distribution. From cases 7 to 12 in Table 7, we randomly pick an integer k out of Poisson distribution with mean $J/25$ for each customer (as the numbers of goods here are all multiples of 25). We then also generate k random numbers out of $U(0, 2)$ for cases 7 to 9, and from Exp(1) for cases 10 to 12 to represent these positive values. For the remaining goods, we still assume the customer assigns a zero value to them. Again, 30 instances are considered for each of the 12 cases.

As shown in Tables 6 and 7, if consumers differ in the number of goods they positively value, offering multiple customized bundles is more profitable than is pure bundling or individual sale. It is also interesting to note that in some cases (e.g., the first case in Table 5, and cases 7 and 8 in Table 7), the pure bundling strategy performs worse than individual sale strategy. The profit improvement of

Table 6 Profit Improvement of Customized Bundling Strategy (Uniform Distributed k)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Customer's reservation price for goods	U(0, 2)	U(0, 2)	U(0, 2)	Exp(1)	Exp(1)	Exp(1)
No. of potential customers I	100	200	500	100	200	500
No. of goods J	50	100	250	50	100	250
Avg. no. of customized bundles offered	8	11	16	6	10	15
Avg. best customized bundling profit found	1,449.4	5,902.7	38,283.7	1,301.2	5,411.7	35,241.7
Avg. best pure bundling profit	1,252.7	5,064.8	32,128.7	1,247.2	5,091.3	31,842.9
Avg. best individual sale profit	1,229.6	4,935.4	31,424.5	906.5	3,658.3	23,108.5
Avg. profit improvement from pure bundling to customized bundling (%)	15.9	16.6	19.2	4.5	6.4	10.7
Avg. profit improvement from individual sale to customized bundling (%)	17.9	19.6	21.8	43.5	47.8	52.5
Avg. duality gap (%)	40.8	40.9	40.6	46.6	46.4	45.5
Avg. computational time	<1 min	<1 min	14 mins	<1 min	<1 min	9 mins
Avg. consumer surplus improvement from pure bundling to customized bundling (%)	19.5	19.5	12.0	24.8	17.8	19.6
Avg. consumer surplus improvement from individual sale to customized bundling (%)	11.4	12.7	12.0	−16.5	−20.0	−25.0
Avg. social welfare improvement from pure bundling to customized bundling (%)	15.7	17.0	16.8	10.0	9.4	13.3
Avg. social welfare improvement from individual sale to customized bundling (%)	15.6	17.3	18.6	13.8	13.9	13.6

Table 7 **Profit Improvement of Customized Bundling Strategy (Poisson Distributed k)**

	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12
Customer's reservation price for goods	U(0, 2)	U(0, 2)	U(0, 2)	Exp(1)	Exp(1)	Exp(1)
No. of potential customers I	100	200	500	100	200	500
No. of goods J	50	100	250	50	100	250
Avg. no. of customized bundles offered	2	3	6	2	3	3
Avg. best customized bundling profit found	112.6	443.5	3,030.5	94.5	365.2	2,635.4
Avg. best pure bundling profit	95.5	389.1	2,816.8	90.2	358.3	2,610.1
Avg. best individual sale profit	104.6	401.3	2,529.5	81.9	299.1	1,855.3
Avg. profit improvement from pure bundling to customized bundling (%)	18.3	14.0	7.6	5.1	2.0	1.0
Avg. profit improvement from individual sale to customized bundling (%)	7.6	10.6	19.8	15.6	22.2	42.1
Avg. duality gap (%)	45.9	47.7	46.8	56.1	57.3	54.4
Avg. computational time	<1 min	1 min	9 mins	<1 min	1 min	13 mins
Avg. consumer surplus improvement from pure bundling to customized bundling (%)	−18.0	−13.3	−4.9	10.5	13.4	4.9
Avg. consumer surplus improvement from individual sale to customized bundling (%)	12.5	12.5	14.2	1.4	−1.8	−8.7
Avg. social welfare improvement from pure bundling to customized bundling (%)	2.6	3.2	3.1	6.1	6.2	2.2
Avg. social welfare improvement from individual sale to customized bundling (%)	7.9	10.8	17.8	8.1	10.2	16.9

customized bundling comes from market expansion (i.e., by serving more customers) and price discrimination (the possibility of offering different bundle sizes and prices for different consumer types). In the setting in which k is very likely to be small (Table 7), pure bundling performs well because the restriction to a single bundle is not an important constraint; if the range of possible bundles becomes larger (Table 6), the greater flexibility of customized bundling becomes more valuable.

These findings lead to the following observations.

Observation 2. Regardless of customer valuation functions, if consumers differ in the number of goods that they positively value, then customized bundling dominates pure bundling and individual sale, and pure bundling is no longer always better than individual sale.

Observation 3. Heterogeneity of number of goods positively valued is more important than heterogeneity of customer valuation function in determining the optimal pricing scheme.

Observation 4. The advantage of customized bundling is small if there is limited variation in the number of goods valued positively.

We also investigate consumer surplus and total social welfare changes in Tables 6 and 7. As shown in the last four rows of these tables, although the consumer surplus improvement is not always guaranteed, total social welfare increases when we adopt customized bundling. The welfare enhancement comes from selling to more customers, which reduces deadweight loss. If there are few distinct customer segments, these losses can be small. This finding suggests that customized bundling is more efficient and profit enhancing for markets that consist of different types of consumers who differ in the number of goods they positively value.

Observation 5. Customized bundling is welfare enhancing when consumers differ in the number of goods they positively value.

4.3. The Impact of Incomplete Information on Optimal Pricing Scheme

If customer reservation prices are known ex ante, then one can easily derive a good customized bundling pricing menu with the proposed algorithm as shown above. However, in reality, sellers seldom know customer reservation prices exactly, and this uncertainty is captured in specifying a demand distribution to represent consumer preferences. The difference between pricing under these two information conditions is interesting for at least two reasons. First, different models rely on different assumptions about knowledge of demand—the classic mixed bundling models (e.g., Spence 1980 and Hanson and Martin 1990) assume knowledge of reservation prices, whereas the information goods literature relies on demand distributions. Second, although it is reasonable for a firm to know the distribution of demand, obtaining exact reservation prices is likely to be costly. The question that arises is how much is better information about consumer demand worth (or conversely, does inexact knowledge of reservation prices really matter).

Our numerical analysis framework enables us to study this question directly. To perform this analysis, we first repeat our earlier customized bundling optimization analysis, drawing random valuations for

Table 8 Relative Profits Under Incomplete Information

	Uniform k	Uniform k	Uniform k	Uniform k	Poisson k	Poisson k	Poisson k	Poisson k
Value distribution	U(0, 2)	U(0, 2)	Exp(1)	Exp(1)	U(0, 2)	U(0, 2)	Exp(1)	Exp(1)
No. of potential customers I	100	200	100	200	100	200	100	200
No. of goods J	50	100	50	100	50	100	50	100
CB2/CB1 (%)	96.1	94.0	96.1	94.3	97.9	99.3	92.4	97.0
PI/CB1 (%)	92.2	91.5	82.7	82.0	84.0	89.1	84.4	87.6
P/CB1 (%)	88.3	83.4	86.0	90.0	81.5	82.0	88.7	95.7
I/CB1 (%)	80.5	83.6	69.6	68.3	83.9	88.6	79.4	79.6

each good and using these values to predetermine an optimal pricing schedule based on 30 instances as shown above (the average profit is described as CB1). Specifically, we choose those most commonly offered bundles in these test runs and use the average prices derived from these test runs. We then draw another 30 sets of actual valuations and apply the predetermined pricing schedule to these new data to assess performance of the pricing schedule (this average profit is designated as CB2). Thus, CB1 indicates the optimal average profit when the pricing schedule is set *after* the values are known (i.e., in the presence of knowledge of the exact reservation prices), whereas CB2 indicates the realized average profit when the pricing schedule is set *before* the values are known (i.e., in the absence of knowledge of the exact reservation prices). We perform similar calculations for several other common pricing schemes: pure bundling and individual sale together (designated as PI); pure bundling alone (P); and individual sale alone (I). We express the comparison of performance as the fraction of CB1 profits (full information, customized bundling) because CB1 profits represent the greatest possible profits of all the schemes we consider.

Table 8 summarizes these comparisons. The "CB2/CB1" row of the table shows that, although incomplete information is clearly costly, the predetermined customized bundling pricing schedule reasonably approximates the optimal average profit achievable if customer valuations are known ex ante. In addition, we can see from the table that other relatively simple pricing schemes (PI, P, and I) suffer more from incomplete information (they are further away from optimal average profit achievable with full information CB1), and customized bundling—even with incomplete information—is still the most profitable pricing scheme. The finding that customized bundling is less sensitive to incomplete information than are other pricing schemes suggests that customized bundling is even more attractive as a pricing scheme when there are uncertainties about consumer valuations.

Observation 6. Under incomplete information, customized bundling still outperforms other simpler pricing schemes. Compared to customized bundling, other relatively simple pricing schemes suffer more from incomplete information.

In the next two subsections, we examine two additional variations of seller characteristics: menu costs and marginal costs per bundle.

4.4. Sensitivity of the Number of Optimal Bundles to the Menu Cost

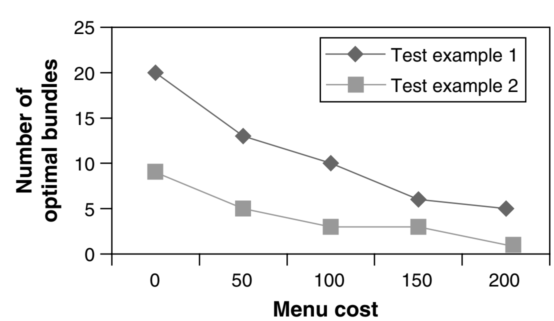
In Figure 1, we present two test cases and compare the optimal number of customized bundles as a function of menu cost. It is clear from Figure 1 that as menu cost increases, the optimal number of bundles offered decreases, as expected. Moreover, even if there is zero cost to maintaining multiple offerings (i.e., zero menu cost), it is not optimal to include all possible bundles; indeed, only a small number of choices will be offered in our solution. For example, in test example 2, our customized bundling strategy chooses to offer the bundles of size 1, 2, 3, 5, 11, 14, 29, 49, and 100 with 13.6% profit improvement from the pure bundling strategy. Note that this is only 9% of the 100 possible bundle choices even though it is theoretically possible in this example for consumers to have k values that span the entire range.

Observation 7. Number of bundles offered decreases as menu cost increases. It is not optimal to offer all possible bundles, and the number of bundles offered represents only a small set of all possible bundles, even with zero menu cost.

4.5. Sensitivity of the Number of Optimal Bundles to the Marginal Bundle Cost

In all previous subsections, we assume negligible costs involved in offering a bundle for sale. In this

Figure 1 Relationship of Number of Bundles to Menu Cost



Notes. Parameters used: Example 1: $I = 1{,}000$, $J = 100$, $B_j = 0.1$; Example 2: $I = 200$, $J = 100$, $B_j = 0.1$. The consumer valuations from both examples are drawn from U(0, 2), and k is uniformly distributed from 0 to J.

Table 9 Relationship of Number of Optimal Bundles and Fixed Marginal Bundle Cost

	$B_f = 0$	$B_f = 10$	$B_f = 20$	$B_f = 30$
No. of customized bundles offered	10	5	4	1
Best customized bundling profit found	829.3	387.4	119.4	1.0
Best pure bundling profit	709.3	346.0	113.7	1.0
Best individual sale profit	699.6	0	0	0
Profit improvement from pure bundling to customized bundling (%)	16.9	12.0	5.1	0
Profit improvement from individual sale to customized bundling	18.5%	N.A.	N.A.	N.A.

Note. $I = 100$, $J = 30$, $M = 0$, k from U(0, J) and customer valuation from U(0, 2) with mean 1.

subsection, we investigate the relationship between the number of optimal bundles and the marginal bundle cost, B_j. We consider two types of marginal bundle costs: marginal reproduction cost for each good, and the fixed bundle cost associated with selling and distributing a bundle, which is independent of the number of goods in the bundle.

We first consider the case in which the marginal reproduction cost for each good is zero but each sale of a bundle incurs a fixed cost (B_f) regardless of the size of the bundle, including an individual sale (Table 9). For instance, this corresponds to the case in which it costs the same amount to package and sell a CD with a single song on it as it does an album CD that is filled to capacity. As shown in all previous subsections, when the marginal bundle cost is negligible, the vendor will be willing to provide several different sizes of bundles. However, when the marginal fixed bundle cost is high, the vendor will have less incentive to sell small size bundles. As is evident from Table 9, as the fixed bundle cost increases, both the number of optimal bundles and the profit decrease. Moreover, if the marginal bundle cost is dominated by the fixed component, individual sale is no longer viable and the improvement of customized bundling over pure bundling decreases. As the marginal bundle cost increases to some point, the vendor will be willing to offer only the pure bundle because this is the only profitable bundle. This explains why, traditionally, it makes sense to bundle hundreds of articles into newspapers for sale, because the transaction costs and distribution costs involved in sending the newspaper to end consumers are relatively high and largely independent of the number of goods.

Observation 8. If the marginal bundle cost is dominated by the fixed component (such as packaging and distribution cost), then as the marginal bundle cost increases, number of bundles offered decreases.

We further study the cases in which marginal bundle cost is increasing with the number of goods in the bundle. Although this could be a reproduction cost, a more likely scenario is that the marginal cost represents royalties to the information good owner. We consider the same conditions as those analyzed in Table 9: $I = 100$, $J = 30$, $M = 0$, k from uniform distribution and customer valuation from U(0, 2). Table 10 reports the results of varying marginal item cost over the full range zero to two. As expected, our results suggest that the profitability of pure bundling is extremely sensitive to *per-item* marginal costs because pure bundling pays the marginal cost on every good, regardless of consumer valuation. This explains why iTunes and other dominant digital music providers will not sell "all" their collections for a fee. Another interesting finding from our analysis is that the number of bundles offered in the customized bundling solution is not very sensitive to *per-item* marginal cost, even if the *per-item* marginal cost is very high. However, as *per-item* marginal cost increases further, there is little incremental benefit over individual sale because the demand for larger bundles becomes negligible (because the price for the large bundle will be too high or the seller will not offer it at all). Also note that if the marginal item cost exceeds the highest possible customer valuation for each good (in this example, 2), there is no pricing solution with positive profit (last column of the table).

Observation 9. Pure bundling is very sensitive to per-item marginal cost. Pure bundling performs significantly worse than individual sale and customized bundling, even with small per-item marginal cost.

Table 10 Profit Improvement vs. Increases in Marginal Bundle Cost with Bundle Size

	$B_j = 0$	$B_j = 0.25j$	$B_j = 0.5j$	$B_j = 0.75j$	$B_j = J$	$B_j = 1.5j$	$B_j = 2j$
No. of customized bundles offered	8	11	8	9	7	7	0
Best customized bundling profit found	872.4	590.8	496.9	343.9	205.0	55.3	0
Best pure bundling profit	755.7	375.1	214.9	54.5	2.4	0	0
Best individual sale profit	744.8	546.5	448.8	297.8	188.6	50.4	0
Profit improvement from pure bundling to customized bundling (%)	15.4	57.5	131.2	530.6	8,270.0	N.A.	N.A.
Profit improvement from individual sale to customized bundling (%)	17.1	8.1	10.7	15.5	8.7	9.6	N.A.

Note. $I = 100$, $J = 30$, $M = 0$, k from U(0, J) and customer valuation from U(0, 2) with mean 1.

Observation 10. The number of bundles offered in the customized bundling is not very sensitive to per-item marginal cost if the per-item marginal cost is smaller than the highest-possible customer valuation.

5. Discussion and Conclusions

As detailed monitoring and customized distribution become feasible and increasingly cost efficient, more flexible pricing schemes become feasible, especially individual sale and micropayments. On the other hand, the power of value extraction through bundling suggests that bundling remains an attractive and profitable strategy even absent benefits from economies of scale. However, there are many common circumstances in which cost structures or consumer preferences make neither individual sale (e.g., high diversity of consumer reservation prices) nor pure bundling attractive (e.g., nonnegligible marginal costs in conjunction with very large J). This suggests considering bundling of subsets of possible goods. One approach, a full mixed bundling solution, offers seller-chosen bundles but suffers from complexity because it potentially involves $2^N - 1$ possible bundles and prices for N goods. Another approach, the mechanism analyzed in this paper—customized bundling—involves the use of customer self-selection to capture some of the benefits of full mixed bundling without the attendant complexity (both for the consumer and the firm). There are a number of potential settings in which a customized bundling solution might prove useful (e.g., cable TV pricing), yet less is known about the properties of this solution than other relevant pricing approaches.

Our paper makes several unique contributions to the literature of customized bundling and bundling in general.

1. We applied nonlinear mixed-integer programming (NLMIP) to the customized bundling pricing problem by determining which bundle sizes to offer and what prices to charge for a monopolist selling a large number of information goods. The flexibility and generalizability of the NLMIP allows us to study any kind of demand distribution or valuation function, which is something not possible with the analytical approach. We are also able to explore the performance of customized bundling, pure bundling, and individual sale under different consumer preferences and conditions, which had also been difficult to accomplish using analytical approaches. This exploration allows us to understand the efficiency loss associated with adopting simple pricing schemes, such as pure bundling or individual sale.

2. We established many interesting results that help us understand the important factors in determining a pricing scheme and help guide the design of a pricing scheme. Our results indicate that heterogeneity of the number of goods positively valued is more important than heterogeneity of the customer valuation function in determining the optimal pricing scheme. Regardless of customer valuation functions, if customers differ in the number of goods they positively value, then customized bundling dominates pure bundling and individual sale, and enhances welfare. This finding has interesting strategic implications because the information about the number of goods a consumer might purchase is relatively easier and cheaper to obtain in practice than is a full customer valuation distribution for each customer and good.

3. We also investigate the efficiency loss due to incomplete information and how this efficiency loss determines the optimal choice of pricing approaches—an observation not well studied in the literature, perhaps because of the difficulty of this analysis without using numerical methods. Our results show that pure bundling and individual sale suffer more from incomplete information, and that customized bundling still outperforms other simpler pricing schemes even when exact consumer valuations are not known ex ante. This suggests that customized bundling pricing is quite robust to incomplete information.

This research not only adds to the information goods pricing literature but also is of practical use in guiding firms on how to bundle and price information goods given their demand and cost structure. For example, our analysis suggests that cable companies and online music sellers (e.g., iTunes) can potentially benefit from customized bundling pricing, although the reasons may be different. In the case of cable market, there is significant customer heterogeneity: Some TV addicts may view large numbers of channels, whereas others only view a very small subset of the channels. The current subscription approach (pure bundling) adopted by cable companies means that many light viewers may be priced out of the market. Yet our analysis also suggests that a move toward a la carte pricing is not necessarily desirable. Our analysis suggests that when consumers differ in the number of goods positively valued, customized bundling would be more profitable, yet socially efficient. As for the music retailer case, given the high positive marginal cost associated with selling each song (which is about $0.50 for royalty to copyright owners), it is clear that pure bundling is not optimal. However, our analysis suggests that the current strategy of individual sale by most music resellers is not optimal either and could be enhanced by incorporating customized bundling.

6. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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